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A Unified View of Particles and Fields in a Simple Model of a Physical System

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We start from the primitive concepts of preparticle and membership relation $\in$ of set theory to obtain the derivative concepts of particle (already introduced in a previous work), field, and the interaction between systems of particles. We have explicitly stated, in addition, what the relationship between a system of particles and the field it produces is in the present model of physical systems. In order to discuss the motion of particles we have analyzed one of the possible definitions of a reference frame.

1. INTRODUCTION

There are three fundamental assumptions that may enter into the formalism or interpretation of physical theories.

Assumption 1 says that matter is made up of elementary constituents which are the invariable and indivisible bricks out of which matter is formed. In a general discussion we prefer the name of elementary constituent instead of elementary particle, because this last term has already a specific connotation in some physical theories, as for instance in quantum field theory.

Assumption 2 stipulates that the interaction between elementary constituents smears out the brick-building structure of matter, giving rise to a kind of continuum or field, and that physical entities as particles are collective excitations of such a continuum (Sakharov, 1967; Misner et al., 1970a).

Assumption 3 says that any physical system is formed by elementary constituents which, whether divisible or not, allow a complete description of

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the system concerned in terms of these elementary constituents and its interactions.

Assumptions 1 and 3 are clearly atomistic hypotheses, though the same kind of hypothesis is hidden in the background of assumption 2. This applies even to physical theories that incorporate continuums as mathematical frameworks, such as occurs for instance in classical and quantum field theories. In these cases, such continuums have their elementary constituents, which are precisely their points. Notice that one cannot define the concept of geometrical continuum without using the concept of geometrical point.

The difference between assumptions 1 and 3, on one hand, and assumption 2 resides mainly in the fact that assumptions 1 and 3 emphasize the elementary constituents of physical systems while assumption 2 underlines the interaction between these elementary constituents, which is frequently described by means of elastic constants. But in each of the assumptions 1, 2, and 3 the existence of elementary constituents is either implicitly or explicitly taken for granted.

Notice that the elementary constituents may change with the physical theory concerned. For instance, the elementary constituents of fluid mechanics are the elements of volume of the fluid which, though containing a large number of molecules are very small in comparison with a characteristic macroscopic volume of the fluid under consideration. The elementary constituents of molecular and solid state physics are the electrons and nuclei, and sometimes the atoms themselves. The elementary constituents of quantum field theory are the elementary particles, experimentally established up to this moment.

The above three assumptions are not incompatible, though separately they may lead to different models of matter. Most frequently they appear "mixed in different proportions," either incorporated into the basis of the theory and/or as interpretative recourses that aid in making intuitive representations of physical systems described by physical theories. For instance, assumption 3 can be harmonized with assumption 1 by adding the specification that elementary constituents of assumption 3 arise from the association of the invariable and indivisible elementary constituents of assumption 1. On the other hand, assumption 3 is also compatible with the idea that the elementary constituents entering into a given physical theory can always be decomposed into new elementary constituents, which in turn enter into a more fundamental theory of matter, and so on and so forth. Yet, this will lead to an infinite regress, which, apart from being a logical defect, weakens the explanatory power of the atomistic hypotheses. In short, assumption 1 cuts off the infinite regress that may be allowed by assumption 3. This is one of the reasons why each time that assumption 3 is considered we should also consider assumption 1.

Now, what do experimental results say about atomistic hypothesis with an infinite regress? First, let us stress that experience cannot give a conclusive answer in the positive sense to this question in a finite lapse of time. This is simply because an infinite number of experiences would be required, one for each step of the infinite regress. However, experience can give an increasing amount of indications in favor of one of the following two possibilities: (i) atomistic hypothesis with infinite regress, or (ii) atomistic hypotheses without infinite regress. Notice that if experience supports assumption (i) it will be a strong reason to give up atomistic models of matter (even if only by the logical defect involved in such a case).

At first sight, one could think that experience supports assumption 3 with an infinite regress, each time towards more elementary constituents of matter. This impression comes from the fact that most of the elementary constituents of matter that have been proposed in the last century, and in the beginning of the present one have been broken in collision experiments where exchange of enough energy takes place. Thus, the supposedly most elementary constituents of matter separate into new particles which have been clearly identified as constituent parts of what were previously considered to be elementary constituents of matter. For instance, molecules decompose into atoms by processes involving exchange of energy relevant to chemistry, i.e., from 1 to ~ 10 eV. Atoms have been broken into electrons and nuclei by collisions where exchange of energy of ~ 10 ev for light atoms and of ~ 10 keV for heavy atoms takes place. Nuclei break down into protons, neutrons, and other particles when exchange of energy roughly around 10 MeV occurs between nuclei. Now, independently of the fact that some of the actually considered elementary particles could appear in the future as composite particles (promising candidates in this sense are the nucleons, which are now suspected to be formed by quarks), something radically new has happened in the experiments of high energy carried out in the last four decades (Heisenberg, 1976; Wichmann, 1967). What has occurred when the attempt has been made to break up the actually considered elementary particles in a collision experiment is that new elementary particles have been created with or without the destruction (which is regulated by the conservation laws) of the originally colliding particles. However, these new elementary particles cannot be considered as forming parts of the particles initially entering into the collision. This entirely new fact points towards the reinforcement of atomistic hypotheses without the above-mentioned infinite regress.

There is at least one difficulty arising from atomistic models of matter. For instance, consider the elementary constituents of the whole universe U and their interactions. One can ask what is the nature of these interactions. Then, if "action at a distance" is disregarded, as is effectively done in modern physics, one is compelled to postulate the existence of a physical entity, say Ψ, through which the interaction between the elementary constituents of U is mediated. Once this has been done, and in order to be consistent with the atomistic hypothesis, one must ask what are the elementary constituents of Ψ. These will be new elementary constituents which were not considered initially when all the elementary constituents of U were given and the attempt was made to explain the interactions existing between them. Accordingly, one starts with all the elementary constituents of U and still there are more elementary constituents of U.

If a part of U (say $u_1 \subset U$) is concerned, then by following a similar argument one arrives at the conclusion that a system u_1 of interacting particles requires the existence of a nonempty system $u_2 \subset U$, such that all the elementary constituents of the physical entity Ψ' mediating the interaction between the constituents of u_1 belong to u_2. Therefore u_1 and u_2 are different sets. In short, when one invokes a system of interacting particles one is really invoking two different systems.

One answer could be that the above paradox arises because we have introduced implicitly the assumption that a particle and the fields that it produces are different entities. Suppose, then, that to avoid the above difficulty one must always consider a particle together with the fields that it produces as a sole physical object. In that case, though we avoid the above paradox, it remains that an elementary constituent of matter invoked in assumption 1 can hardly be identified with such a complex entity as an elementary particle together with the fields that it produces. In particular, the spatially extended character of fields fits well into the idea that fields can be considered as formed by an infinite number of field oscillators. Each one of these oscillators is an entity more simple than the field to which it belongs.

Accordingly, it may be of interest to propose a theory that is probably too simple to be true, but founds the concepts of particle, field, and interaction between particles on only two primitive concepts that are of a simple nature and are intuitively clear. These are the concepts of preparticle and membership relation of set theory. Furthermore, we hope that this theory will be free from paradoxes of the type discussed above.

To this end we develop further a theory of time formulated in a previous paper (García-Sucre, 1975). Therein was postulated the existence of unchangeable physical objects without internal structure which were called preparticles. Here preparticles are considered to be the basic ingredient of the world. Furthermore, preparticles cannot interact. It is more precise to state that interaction between particles will be shown here to be a derivative concept not defined for any pair of preparticles or a pair consisting of a preparticle and a particle. In short, this is the way we propose to circumvent the above-discussed paradox. Namely, the elementary constituents of matter are preparticles that are different from particles, and only particles can interact.

From the point of view of atomistic models, our theory can be considered as an atomistic model of matter with a cutoff of the above-discussed infinite regress. The cutoff occurs at the level of preparticles. Furthermore, particles either elementary or composite have the common trait of having the preparticles as basic ingredients.

The fact that preparticles do not interact imposes the restriction that preparticles cannot be detected, since it is hard to conceive how a physical entity can be detected without interacting with it. As we will see in Section 2, this restriction does not apply to particles since these entities can interact in our model. Hence, particles may have in our model a physical interpretation, or differently stated, there can be given a semantic rule of reference (Bunge, 1967) relating them to the actual particles that have been detected experimentally. Yet, we do not describe in our model the detailed properties of these entities, but rather their general traits.

2. PARTICLES AND FIELDS

In a previous paper (García-Sucre, 1975) the existence of a denumerably infinite set $B \equiv \{\alpha_i \mid i \in N\}$ was postulated, where N is the set of all natural numbers. We have called this the base set of all physical systems to which all the preparticles α_i belong. Notice that B has no structure, its only property being that of having all the preparticles as unique members.

Relations of temporal and spatial order between occurrences are among the more fundamental relationships in physics (Russell, 1969). We have shown (García-Sucre, 1975) that time and temporal order can be obtained as a derivative concept starting uniquely from the concepts of preparticle and membership relation $\in$ of set theory. In so doing, we have been guided by a very interesting property of orders demonstrated by Hessenberg (1906), Kuratowski (1921), and Fraenkel (1925), and clearly discussed by Fraenkel and Bar-Hillel (1958). To see what this property consists of let us first recall some elementary definitions of set theory (see for instance Fraenkel, 1961a).

The *power-set* $P(Q)$ of a set Q is the set whose members are all the subsets of Q. For instance, consider a set $Q = \{a, b\}$. Then $P(Q) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$.

A *chain* is a set C of sets x such that for any two different members x and x' of C, we have either $x \subset x'$ or $x' \subset x$ (Fraenkel and Bar-Hillel, 1958). To given an example of a chain consider a set $C = \{x, x'\}$ with $x = \{\alpha_1\}$ and $x' = \{\alpha_1, \alpha_4, \alpha_5\}$. In this case C is a chain since we have $x \subset x'$.

Given a set Q, a *maximal chain* M of subsets of Q is defined by the following properties: (a) $M \subseteq P(Q)$, (b) M is a chain, and (c) if M' satisfies (a) and (b), and $M \subseteq M'$, then $M = M'$ (Fraenkel and Bar-Hillel, 1958).

Consider the set $Q = \{\alpha_1, \alpha_2\}$. Then $P(Q) = \{\varnothing, \{\alpha_1\}, \{\alpha_2\}, \{\alpha_1, \alpha_2\}\}$ and the maximal chains included in $P(Q)$ are $M_1 = \{\varnothing, \{\alpha_1\}, \{\alpha_1, \alpha_2\}\}$ and $M_2 = \{\varnothing, \{\alpha_2\}, \{\alpha_1, \alpha_2\}\}$.

According to Hessenberg (1906), Kuratowski (1921), and Fraenkel (1925) ordered sets can be defined in the following way:

A set Q is said to be *orderable* if a maximal chain of subsets of Q exists. Furthermore, for every set Q there exists at least a maximal chain M which defines an order in Q; i.e., a rule giving rise to a relation $\prec$ between the members of Q fulfilling the well-known properties of connectivity, assymetry, irreflexivity, transitivity, and substitutivity (Fraenkel, 1961b).

The way in which a given maximal chain M included in $P(Q)$ orders Q can be briefly described and is as follows: For any two different members s and s' of Q there exists a $x \in M$ such that either $s \in x$ and $s' \notin x$ or $s' \in x$ and $s \notin x$. The first and second cases correspond $s \prec s'$ and $s' \prec s$ respectively. In this way the concept of order can be reduced to the membership relation $\in$ of set theory.

Let us give an example. Consider a finite set $Q_a = \{s_1, s_2, s_3\}$ and its corresponding power-set $P(Q_a) = \{\varnothing, \{s_1\}, \{s_2\}, \{s_3\}, \{s_1, s_2\}, \{s_1, s_2\}, \{s_1, s_3\}, \{s_2, s_3\}, \{s_1, s_2, s_3\}\}$. Then, all the maximal chains included in $P(Q_a)$ are given by $M_1 = \{\varnothing, \{s_1\}, \{s_1, s_2\}, \{s_1, s_2, s_3\}\}$, $M_2 = \{\varnothing, \{s_1\}, \{s_1, s_3\}, \{s_1, s_2, s_3\}\}$, $M_3 = \{\varnothing, \{s_2\}, \{s_1, s_2\}, \{s_1, s_2, s_3\}\}$, $M_4 = \{\varnothing, \{s_2\}, \{s_2, s_3\}, \{s_1, s_2, s_3\}\}$, $M_5 = \{\varnothing, \{s_3\}, \{s_1, s_3\}, \{s_1, s_2, s_3\}\}$ and $M_6 = \{\varnothing, \{s_3\}, \{s_2, s_3\}, \{s_1, s_2, s_3\}\}$. It can easily be seen that the above M_i's fulfill the properties (a), (b), and (c) given above as characterizing completely a maximal chain. Furthermore, M_1, M_2, M_3, M_4, M_5, and M_6 induce ordering in Q which respectively give rise to the ordered sets (s_1, s_2, s_3), (s_1, s_3, s_2), (s_2, s_1, s_3), (s_2, s_3, s_1), (s_3, s_1, s_2), and (s_3, s_2, s_1). We have used the parentheses ( ) to denote ordered sets, instead of the curly brackets { }, which, following the usual notation, we have reserved for plain sets.

To illustrate the way in which the above maximal chains order the set $Q_a = \{s_1, s_2, s_3\}$, consider the maximal chain $M_3 = \{\varnothing, \{s_2\}, \{s_1, s_2\}, \{s_1, s_2, s_3\}\}$. For this case we have that there exists a $x \in M_3$ $(x = \{s_2\})$ such that $s_2 \in x$ and $s_1, s_3 \notin x$. Then, according to what we have already said above (in relation to the way in which a given maximal chain M included in $P(Q)$ orders Q) it follows that $s_2 \prec s_1$ and $s_2 \prec s_3$. In the same way there exists a $y \in M$ $(y = \{s_1, s_2\})$ such that $s_1 \in y$ and $s_3 \notin y$ and thus $s_1 \prec s_3$. Therefore, we finally obtain that M_3 orders $Q_a = \{s_1, s_2, s_3\}$ yielding the ordered set (s_2, s_1, s_3).

In the present work we will use the concept of *partial chain* rather than maximal chains, which will allow a more flexible classification of the subsets of $P(Q)$. For us, a partial chain is any subset of $P(Q)$. Accordingly, the concept of partial chain covers the concept of chain as a particular case, which in turn covers the concept of maximal chain. In general, in a partial chain $\tilde{C}$ there exists a set $C \subseteq \tilde{C}$ which is a chain, whether empty or not. In this connection consider a set $G_i = \{a^i(x) \mid x \in X \text{ and } a^i(x) \in P(Q)\}$, where $X \subseteq R$ and $i \in N$, and R stands for any set of indexes with the power of the continuum. Then we call α *state* of G_i any set

$$s^i(x) = a^i(x) - \bigcup_{x' \in X'(x)} a^i(x') \tag{2.1}$$

where the $a^i(x')$'s with $x' \in X'(x)$ are all the members of G_i such that $a^i(x) \nsubseteq a^i(x')$ (García-Sucre, 1975). Furthermore, if some or all of the members of G_i are ordered by the proper inclusion relation $\subset$, then some or all of the α states of G_i are ordered by the following rule: Given two α states of G_i

$$s^i(x) = a^i(x) - \bigcup_{x' \in X'(x)} a^i(x')$$

and

$$s^i(y) = a^i(y) - \bigcup_{y' \in X'(y)} a^i(y')$$

then one has $s^i(x) \prec s^i(y)$ iff $a^i(x) \subset a^i(y)$. We will denote by $\sum(G_i)$ the set of all the α states of G_i. Chains induce partial ordering of the subsets of Q, and completely order a special set of subsets of Q, namely, the set of α states of the chain concerned. For instance, consider again the set $Q_a = \{s_1, s_2, s_3\}$ and the chain $C_1 \subset P(Q_a)$ given by $C_1 = \{\{s_1\}, \{s_1, s_2, s_3\}\}$. Then C_1 partially orders Q, since the order induced in Q by C_1 is given by $s_1 \prec s_2$ and $s_1 \prec s_3$; however, nothing has been said in relation to the order between s_2 and s_3. Nevertheless, from equation (2.1) it follows that the α states of C_1 are given by $\{s_1\} = \{s_1\} - \varnothing$ and $\{s_2, s_3\} = \{s_1, s_2, s_3\} - \{s_1\}$, and according to the above ordering rule we have $\{s_1\} \prec \{s_2, s_3\}$ since $\{s_1\} \subset \{s_1, s_2, s_3\}$. Therefore C_1 completely orders the set $\sum(C_1)$ whose members are the α states of C_1.

On the other hand, a partial chain $\tilde{C}_2 \subset P(Q_a)$ given by $\tilde{C}_2 = \{\{s_1\}, \{s_1, s_2\}, \{s_3\}\}$ is such that the chain $C_2 = \{\{s_1\}, \{s_1, s_2\}\}$ is properly included in $\tilde{C}_2$. Accordingly, $\tilde{C}_2$ orders partially Q_a, and in contrast with the last case, $\tilde{C}_2$ also orders partially the set $\sum(\tilde{C}_2)$.

What we have said before suggests that to obtain from the plain set $B = \{\alpha_i \mid i \in N\}$ (whose members are all the preparticles) a set structured by orders relations, the simplest candidate to be considered is the power set $P(B)$.

We then introduce the following definitions:

Definition 1. Given the base set B, whose members are all the preparticles, any subset of $P(B)$ represents a particle (García-Sucre, 1975).

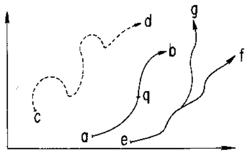

Fig. 1. *ab*, *cd*, *ef*, and *eg* are arcs of curve whose elements are rational points. *ab* and *cd* illustrate the case of evolving particles such that every point of each arc is an α state of the evolving particles concerned. The arcs *ef* and *cg* together represent a nonevolving particle.

According to this definition the union and the intersection of two particles are also particles. In order to avoid exceptions we will agree to call the empty set $\varnothing$, which is obviously a subset of B, the empty particle.

We distinguish two types of particles (García-Sucre, 1975): those particles represented by chains, which we call *evolving particles*; and those represented by subsets of $P(B)$, which are not *chains, and which we call nonevolving particles.*

According to these definitions, given an evolving particle p, then the set $\sum(p)$ of α states of p can be interpreted as the whole history of the particle and each α state as a stage of this history. This interpretation does not apply to the case of a nonevolving particle p', since an order cannot be given for some or all the α states of p'. However, there may exist nonevolving particles $\tilde{p}$ such that a subset of $\sum(\tilde{p})$ can be interpreted as a partial history of $\tilde{p}$.

To make clearer the difference between evolving and nonevolving particles let us discuss the following analogy. Suppose we have an evolving particle p. Then the set $\sum(p)$ will be a completely ordered set of α-states of p. If each α state is interpreted as a stage of the history of p, then by counting the α states belonging to $\sum(p)$ which are between given pairs of α states of p, we can establish a clock that measures proper-time intervals within p. According to what we have said above in connection with nonevolving particles it follows that within a nonevolving particle proper-time intervals cannot be defined for every pair of α states of such a particle.

Therefore, if one assumes that within a particle it is always possible to define proper-time intervals between any pair of events of that particle, then all the existing particles are evolving particles. However, if this is not the case, then nonevolving particles also exist.

To illustrate Definition 1 let us consider the following representations of particles. Call *ab* the set of points belonging to the arc of curve shown in Figure 1, such that their coordinates are rational numbers. The set *ab* is a reinterpretation of an evolving particle whose elements are all the initials of *ab* determined by every point of the arc *ab*. An initial of a set Q is any subset Q' of Q such that Q' contains together with any $s_0 \in Q'$ all $s \in Q$ for which $s \prec s_0$ in Q (Franekel, 1961c). Then, every point of *ab* is an α state of the evolving particle represented by *ab* and thus is an element of $\sum (ab)$. The arrowhead on each curve specifies the way in which the points of each curve are ordered. Accordingly, in Figure 1 every point q belonging to *ab* is an α state of *ab*. Furthermore, the initial *aq* determined in *ab* by q is an element of *ab*. Another example is provided by the points belonging to the discontinuous curve *cd* appearing in Figure 1. This represents an evolving particle whose elements are all the initials of *cd* determined by rational points belonging to *cd*. Again, each rational point of *cd* is an α state of *cd*, i.e., a member of $\sum (cd)$. There appears a third example in Figure 1 corresponding to a nonevolving particle. Its elements are represented by the initials of the arcs *ef* and *eg* determined by every rational point of either *ef* or *eg*. More generally, a nonevolving particle would appear on the figure as a set of points and/or regions of the plane that are either only partially ordered, or not ordered at all. Each of these points and regions would represent an α state of the particle concerned.

Definition 2. We call an arbitrary set of particles a *physical* system (García-Sucre, 1975).

Definition 3. Given a physical system S and an α state $s^i(x)$ of a particle p_i belonging to S, we call *complex* $\sigma(s^i(x); S)$ any ordered pair $(s^i(x); \prod_x^i (S))$, where $\prod_x^i (S) \subseteq S$ and for any $p \in \prod_x^i (S)$ there exists at least one $s \in \sum (p)$ fulfilling $s \cap s^i(x) \neq \varnothing$. We call $s^i(x)$ and $\prod_x^i (S)$ the *center* and the $\prod$ *set* of the complex $\sigma(s^i(x); S)$, respectively.

Let the lines of Figure 2 be formed by rational points and assume that the lines appearing on this figure represent all the particles belonging to a given physical system S. Then, in Figure 2 there are as many complexes

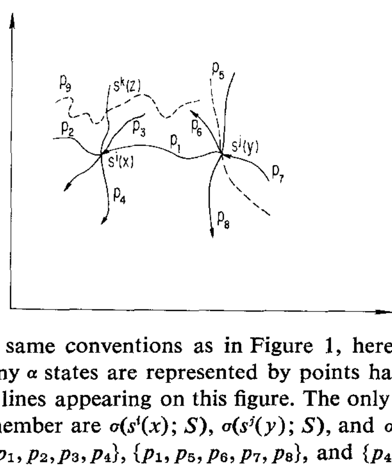

Fig. 2. Following the same conventions as in Figure 1, here are illustrated as many complexes, and as many α states are represented by points having rational coordinates and belonging to the lines appearing on this figure. The only complexes having $\prod$ sets with more than one member are $\sigma(s^i(x); S)$, $\sigma(s^j(y); S)$, and $\sigma(s^k(z); S)$, and the corresponding $\prod$ sets are $\{p_1, p_2, p_3, p_4\}$, $\{p_1, p_5, p_6, p_7, p_8\}$, and $\{p_4, p_9\}$, respectively.

represented as there are α states represented by rational points belonging to the lines appearing in this figure. The only complexes having associated with them $\prod$ sets with more than one particle are those given by $\sigma(s^i(x); S)$, $\sigma(s^j(y); S)$, and $\sigma(s^k(z); S)$. Because of Definition 3 and Figure 2 the corresponding $\prod$ sets are given by $\prod_x^i(S) = \{p_1, p_2, p_3, p_4\}$, $\prod_y^j(S) = \{p_1, p_5, p_6, p_7, p_8\}$ and $\prod_z^k(S) = \{p_4, p_9\}$.

Figures 1 and 2 were constructed in such a way that every α state of the particles represented there is a set containing one preparticle only. However, according to our definition of particle, an α state of a particle can be a set of more than one preparticle. Such α states we will call *composite* α *states*. Figure 3 represents some particles of a system S with the same conventions as in Figures 1 and 2; i.e., points of the plane with rational coordinates represent preparticles and every continuous or discontinuous curve of rational points with an arrowhead represents an evolving particle. In addition, we represent in this figure particles with some, or all, of their α states composite. A composite α state is represented by enclosed regions whose internal rational points are the preparticles belonging to the represented α state. We make the convention that the closed curves serving to specify sets of preparticles forming α states do not represent any particle of S. The particle C_1 in the above figure is such that there is no particle belonging to S appearing in the interior of any one of its composite α states. We will say that such composite states are *internally disconnected* in S. Using this terminology, some of the α states of the particle C_m represented in Figure 3 are internally disconnected. On the other hand, the α states of the particles C_i and C_k are sets of one preparticle.

The *connection between particles or between complexes* is given by the set of all particles such that some of their α states have a nonempty intersection

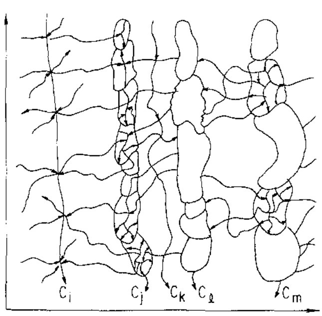

Fig. 3. Following the same conventions as in Figures 1 and 2 here are represented some particles of a system S with some or all of their α states composite. A composite α state is represented by enclosed regions whose internal points with rational coordinates are the preparticles belonging to the represented α state. The concept of internally disconnected α state is also illustrated by those regions without internal lines.

with some of the α states of the connected particles, or with the α states that are centers of the connected complexes.

We will speak of two particles as *directly connected* when the corresponding connection is a nonempty set. Furthermore, we will consider two particles as *indirectly connected* when, even though the corresponding connection is an empty set, there exists a set P of particles to which our two particles belong, and the members of P form a sequence of particles such that two consecutive members are directly connected. Otherwise, we will speak about *disconnected particles*. In a parallel way, definitions of *directly connected, indirectly connected*, and *disconnected* complexes can be introduced.

Definition 4. We say that two particles p_i and p_j are *similar* iff there exists a one-to-one mapping, say ψ, between the sets $\sum(p_i)$ and $\sum(p_j)$ which preserves the order of the α states of both particles, and such that two corresponding α states are equivalent sets of preparticles. We call ψ a similarity mapping between $\sum(p_i)$ and $\sum(p_j)$.

Notice that according to the above definition the mapping ψ preserves the order in $\sum(p_j)$ and $\sum(p_j)$, including the case where $\sum(p_i)$ and $\sum(p_j)$ are only partially ordered sets.

Definition 5. Two complexes $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$ are similar, and we denote this as $\sigma(s^i(x); S) \sim \sigma(s^j(y); S)$, iff there exists a one-to-one mapping φ between $\prod_x^i(S)$ and $\prod_y^j(S)$ fulfilling the following conditions:

(a) If $p_i \in \prod_x^i(S), p_j \in \prod_y^j(S)$ and $p_i \overset{\psi}{\longleftrightarrow} p_j$, then p_i and p_j are similar particles.

(b) There exists a similarity mapping $\sum(p_i) \overset{\psi}{\longleftrightarrow} \sum(p_j)$ (Definition 4) which puts into correspondence $s^i(x)$ and $s^j(y)$.

(c) If $p_i, p_i' \in \prod_x^i(S)$, $p_j, p_j' \in \prod_y^j(S)$, $\sum(p_i) \overset{\psi}{\longleftrightarrow} \sum(p_j)$, and $\sum(p_i') \overset{\psi}{\longleftrightarrow} \sum(p_j')$, then the intersections between the α states of p_i with those of p_i' are equivalent to the intersections between the ψ-corresponding α states of p_j with those of p_j'.

Notice that according to the above definition one of the consequences of $\sigma(s^i(x); S) \sim \sigma(s^j(y); S)$ is that $s^i(x)$ and $s^j(y)$ must contain the same number of preparticles.

Let us give an example illustrating definitions 4 and 5. Figure 4 represents the complexes $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$, where $S = \{p_1, p_2, \ldots, p_{10}\}$. Particles p_i ($i = 1, 2, \ldots, 10$) are represented with the same conventions as the particles appearing in Figure 1. Therefore, each rational point on each arrow appearing in Figure 4 is an α state of the particle represented by the arrow under consideration; i.e., the α states of any $p_i \in S$ are sets to which belong only one preparticle. All the particles appearing in the figure are similar to each other (see Definition 4), given that for any two particles $p_i, p_j \in S$,

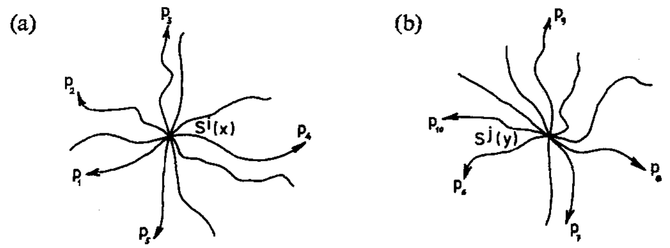

Fig. 4. $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$ are similar complexes. In the case illustrated above all the particles entering into both complexes are similar to each other. The $\prod$ sets $\prod_x{}^i (S)$ and $\prod_y{}^j (S)$ of these two complexes are both sets of five evolving particles.

there exists a one-to-one mapping between $\sum (p_i)$ and $\sum (p_j)$ preserving the order in both sets, and the α states that are in correspondence are sets of the same number of preparticles. Furthermore, the sets $\prod_x^i (S)$ and $\prod_y^j (S)$ corresponding to $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$ are equivalent sets of particles (both are sets of five particles). In addition, for two given $p_i \in \prod_x^i (S)$ and $p_j \in \prod_y^j (S)$ there always exist a similarity mapping $\sum [p_i] \overset{\psi}{\leftrightarrow} \sum [p_j]$ that puts into correspondence the α states $s^i(x)$ and $s^j(y)$. Therefore, according to Definition 5 the complexes $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$ are similar.

Definition 6. Given a physical system S we call a *structure-set of* S the set $\sum \sum (S)$ whose members are all the complexes of S.

Definition 7. The field f produced by a physical system S is given by the quotient set $\sum \sum (S)/\backsim$. We call a *point of* f any equivalence class $a \in \sum \sum (S)/\backsim$.

In Figure 5 we give an example of a field produced by a system S. Figure 5a represents a system S whose members are all of the evolving particles appearing therein as arrows. These are represented by following the same conventions as for Figure 1. In a similar way as occurs in the case of Figure 4, every rational point lying on any arrow is an α state of a particle belonging to S. The structure set of S, $\sum \sum (S)$, is a set whose members are all the complexes that can be constructed from the particles belonging to S. (See Definition 6.) Thus, there are as many complexes as rational points belonging to the arrows appearing on the figure. $\sum \sum (S)$ is then a denumerably infinite set.

Furthermore, there are three types of complexes belonging to $\sum \sum (S)$: complexes for which there are, respectively, four, three, and one evolving particles passing over the center of the complex. With the same argument already given in the description of Figure 4, it follows that all the complexes consisting of four arrows are similar. The same applies to complexes for

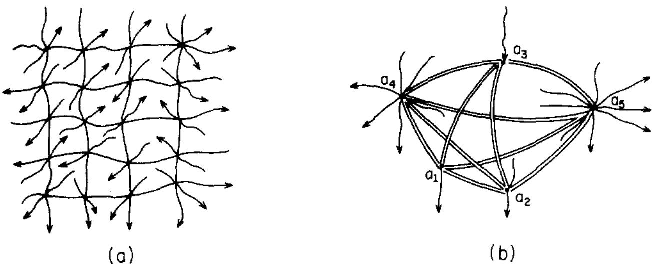

Fig. 5. In (a) is represented a system S whose members are all of the evolving particles appearing therein as arrows. In (b) the field $f = \sum\sum(S)/\backsim$ produced by S is illustrated. The field f has in this example only five points denoted as a_1, a_2, a_3, a_4, and a_5. Each such point is an equivalence class of complexes of the system S. The double-line connections between the points of f stand for all the particles connecting complexes belonging, respectively, to the equivalence classes a^i and a^j with $i, j = 1, 2, \ldots, 5$.

which three arrows pass over the center of the complex. The corresponding equivalence classes with respect to the similarity relation between complexes $\backsim$ (see Definition 5), are represented in Figure 5b by choosing an arbitrary representative of each equivalence class. These two equivalence classes are denoted as a_5 and a_4 in Figure 5b.

For complexes such that only one arrow passes over the center of the complex, there are three different equivalence classes: namely, equivalence classes of complexes such that the center of each complex is either the first, intermediate, or the last α state of the unique arrow involved in the complex. In Figure 5b these equivalence classes are labeled a_1, a_2, and a_3 respectively.

For any two evolving particles p_i and p_j there does not exist a similarity mapping $\sum(p_i) \overset{\psi}{\longleftrightarrow} \sum(p_j)$ that puts into correspondence a first member of $\sum(p_i)$ with either an intermediate or last member of $\sum(p_j)$. As a result we have the three equivalence classes a_1, a_2, and a_3.

The double-line connections appearing on this figure stand for all the particles connecting complexes belonging, respectively, to the equivalence classes a_i and a_j with $i \neq j$ and $i, j = 1, \ldots, 5$. The field f produced by the system S (Definition 7) represented in the Figure 5a is then the set $\sum\sum(S)/\backsim$ $= \{a_i \mid i = 1, \ldots, 5\}$ of equivalence classes a_i illustrated in Figure 5b. Furthermore, each a_i is a point of f (Definition 7).

The above definition of field fits well into the idea of geometrizing fields, typical of general relativity and more generally of geometrodynamics (Wheeler, 1962). In these theories, fields can always be interpreted in terms of the geometry of space-time. Although we do not analyze the concept of space-time in the present paper [this will be studied in a forthcoming paper (García-Sucre, 1977)], we can roughly say that we will consider space-time as a kind of global field. Here, given a physical system S, the corresponding field contains information about the structure of each point of the field and the way in which these points are connected, all giving the topology of the field. Yet, the above definitions allow for change of topology in a way depending on the particles entering in the system S, which produces the field in question. The way in which topology may change in our model will be stated below, where the concept of interaction between systems of particles will be introduced (see Definition 9). This feature of our model opens a possibility to solve a crucial problem raised by Wheeler (1962), namely, that a description of the spin in the framework of geometrodynamics requires that the topology of space-time could change. However, such a change cannot be produced by any topological transformation, i.e., a biunique and continuous transformation.

Notice that according to Definition 7 each point of a field is such that the complexes belonging to it have a structure different from the structure of the complexes belonging to the remaining points of the field (however slight or strong the difference may be). This feature allows for the complete characterization of each point of a field in relation to the remaining points of the same field.

Another reason to call field the entity introduced in Definition 7 is that, in the scheme of geometrizing fields, this definition permits an easy characterization of separations between complexes of a field $\sum\sum(S)/\backsim$. Namely, a time separation between two complexes $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$ is essentially given by an evolving particle p belonging to S, such that the first and last α state of p have a nonempty intersection with $s^i(x)$ and $s^j(y)$, respectively. The sense of the time separation being given by the order of the set $\sum(p)$ of α states of p. A measure of the above separation could be the number of α states belonging to $\sum(p)$. On the other hand, a space separation between the above complexes is given by a sequence of complexes fulfilling the following: (i) Consecutive complexes are directly connected by an evolving particle; (ii) the sense of the connections alternates from one connection to the neighboring one in the sequence; and (iii) the evolving particles making up the connections are similar and have only two α states (see Figure 6).

A measure of a space separation could be the number of complexes entering in the above-mentioned sequence.

In general, two complexes will be connected in so many different ways that different kinds of separations exist between them. Moreover, the connections and separations between points x and x' belonging to a field are given by all the connections and separations between complexes σ_i and σ_j belonging to x and x', respectively (Figure 5).

To analyze further the problem of the connections and separations between points of a field let us introduce the following concepts:

A partial complex $\tilde{\sigma}(s^i(x); S)$ *of a given complex* $\sigma(s^A(x); S) =$

$(s^i(x); \prod_x^i S))$ is any ordered pair $(s^i(x); \prod_x^i (S))$ for which $\tilde{\prod}_x^i (S) \subseteq \prod_x^i (S)$.

A partial field $\tilde{f}$ of a field f is any set of equivalence classes $\tilde{x}_i$ (with respect to $\backsim$, see Definition 5) of partial complexes obtained from the complexes belonging to the points of f. We call any $\tilde{x}_i \in \tilde{f}$ a point of $\tilde{f}$.

According to the above definitions given a complex $\sigma(s^i(x); S)$, then any complex $\sigma'(s^i(x); S')$, where $S' \subseteq S$, is a partial complex of $\sigma(s^i(x); S)$. Similarly, the field f' produced by S' is a partial field of the field f produced by S. However, given a partial complex $\tilde{\sigma}(s^i(x); S)$ and a partial field $\tilde{f}$ of $\sigma(s^i(x); S)$ and f, respectively, there does not always exist a system $S' \subseteq S$ for which we have $\tilde{\sigma}(s^i(x); S) = \sigma(s^i(x); S')$ and $f' = \tilde{f}$. For instance, consider the case $S = \{p_i, p_j\}$ and $\sigma(s^i(x); S) = (s^i(x); \{p_i\})$. Then, there is no $S' \subseteq S$ such that $\sigma(s^i(x); S') = \tilde{\sigma}(s^i(x); S) = (s^i(x); \varnothing)$. This is because $\sigma(s^i(x); S) = (s^i(x); \{p_i\})$ implies $s^i(x) \in \sum (p_i)$. Furthermore, any $S' \subseteq S$ for which there exists a complex $\sigma(s^i(x); \prod_x^i (S'))$ must be such that at least a particle, say p', belonging to S' fulfills $s^i(x) \in \sum (p')$. However, in such a case we have $p' \in \prod_x^i (S')$ and therefore $\prod_x^i(S') \neq \varnothing$.

An *evolving-particle-stretch* of a given evolving particle p_i is any evolving particle p_i^k such that the set of α states $\sum (p_i^k)$ is a subset of consecutive members of $\sum (p_i)$.

For instance, $p_i^k = \{\{\alpha_1\}, \{\alpha_1, \alpha_2\}\}$ is an evolving-particle-stretch of $p_i = \{\{\alpha_1\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_2, \alpha_3\}\}$ since $\sum (p_i) = (\{\alpha_1\}, \{\alpha_2\}, \{\alpha_3\})$ and $\sum (p_i^k) = (\{\alpha_1\}, \{\alpha_2\})$. Another example of an evolving-particle-stretch of p_i is provided by p_i itself since $\sum (p_i) \subseteq \sum (p_i)$.

An evolving channel EC is a set of evolving particles such that there exists a $p_c \in \mathrm{EC}$ so that any $p_i \in \mathrm{EC}$ is an evolving-particle-stretch of p_c.

An example of evolving channel is provided by the set $\mathrm{EC}_1 = \{p_1, p_2, p_3\}$, where $p_1 = \{\{\alpha_1\}\}$, $p_2 = \{\{\alpha_2\}\}$, and $p_3 = \{\{\alpha_1\}, \{\alpha_1, \alpha_2\}\}$. Similarly, $\mathrm{EC}_2 = \{p_1, p_3\}$ and $\mathrm{EC}_3 = \{p_2, p_3\}$ are also evolving channels.

Two evolving channels EC and EC′ are *similar* if for every particle $p_i \in \mathrm{EC}$ there exists a $p_i' \in \mathrm{EC}'$ similar to p_i, and conversely (see Definition 4).

Two evolving channels EC and EC′ *cross each other* if there exists at least two particles p_i and p_i' belonging, respectively, to EC and EC′ such that $s \in \sum (p_i)$, $s' \in \sum (p_i')$ and $s \cap s' \neq \varnothing$.

We say that *a point $\tilde{x} \in \tilde{f}$ is in the evolving channel EC* if there exists at least one $\tilde{\sigma}(s^i(x); \tilde{\prod}_x^i (S)) \in \tilde{x}$ for which $s^i(x) \in \sum (p)$ and $p \in \mathrm{EC}$.

Two evolving channels EC and EC′ are *immediately connected* if (i) every $\tilde{x}$ in EC is connected to only one $\tilde{x}'$ in EC′, and conversely; (ii) each such connection is a set of only one evolving particle; and, (iii) each such particle has only two α states, which are, respectively, the centers of two partial complexes, one of them belonging to $\tilde{x}$ and the other to $\tilde{x}'$. In this case we will also say that the points $\tilde{x}$ and $\tilde{x}'$ are immediately connected.

A set $\overline{\mathrm{EC}} = \{\mathrm{EC}_i \mid i \in I \subset N\}$ of evolving channels *exactly covers a partial field* $\tilde{f}_g$ of the field f_g produced by a system S_g if (i) for any $p \in \mathrm{EC}_i \in \overline{\mathrm{EC}}$ there exists at least one $\tilde{\sigma}(s^i(x); \tilde{\prod}_x^i (S_g)) \in \tilde{x} \in \tilde{f}_g$ such that $p \in \tilde{\prod}_x^i (S_g)$; (ii) every $\tilde{x} \in \tilde{f}$ is in a $\mathrm{EC}_i \in \overline{\mathrm{EC}}$; and (iii) every $p_j \in \tilde{\prod}_y^j (S_g)$ [where $\tilde{\prod}_y^j (S_g)$ is the $\prod$ set of a partial complex belonging to a point of $\tilde{f}_g$] that does not belong to any $\mathrm{EC}_i \in \overline{\mathrm{EC}}$ immediately connects two point of $\tilde{f}_g$.

If a set $\overline{\mathrm{EC}} = \{\mathrm{EC}_i \mid i \in I \subset N\}$ covers exactly $\tilde{f}_g$ and every $\mathrm{EC}_i \in \overline{\mathrm{EC}}$ is immediately connected to either n or $(n - 1)$ evolving channels of $\overline{\mathrm{EC}}$, then we will say that $\tilde{f}_g$ has an *order of immediate connection equal to n and a boundary of order n* $-$ 1. If, on the other hand, every $\mathrm{EC}_i \in \overline{\mathrm{EC}}$ is immediately connected to n evolving channels belonging to $\overline{\mathrm{EC}}$, then we will say that $\tilde{f}_g$ *has no boundaries* and has an order of immediate connection equal to n.

Definition 8. Given a system S_g and the field f_g produced by S_g, we say that a partial field $\tilde{f}_g$ of f_g is a *reference frame* in f_g if there exists a set $\overline{\mathrm{EC}} = \{\mathrm{EC}_1, \mathrm{EC}_2, \ldots, \mathrm{EC}_i, \ldots\}$ of evolving channels fulfilling the following conditions:

(i) $\overline{\mathrm{EC}}$ covers exactly $\tilde{f}_g$, and in any sequence of points of $\tilde{f}_g$ in different EC's of $\overline{\mathrm{EC}}$ such that two neighboring points are immediately connected the sense of the connections alternate.

(ii) The α states of the evolving particles belonging to any $\mathrm{EC}_i \in \mathrm{EC}$ are equivalent sets of preparticles.

(iii) If $\mathrm{EC}_i, \mathrm{EC}_j \in \overline{\mathrm{EC}}$ then EC_i is similar to EC_j.

(iv) If $\mathrm{EC}_i, \mathrm{EC}_j \in \overline{\mathrm{EC}}$ then EC_i does not cross EC_j.

(v) $\tilde{f}_g$ has an order of immediate connection equal to n and either has no boundaries or has a boundary of order $n - 1$.

Suppose we are concerned with a system S_g to which belongs a large number of particles of many different kinds, connected in such a way that the connection between two α states consists of a large number of particles. Then, the corresponding structure set $\sum\sum(S_g)$ will be rich in different types of complexes. The field $\sum\sum(S_g)/\backsim$ may then have a large number of points and for almost any pair of these points there are many particles of S connecting them.

We may now ask: How can we define time and space separations in such a way that for two points x and x' of f_g it is possible to assign unambiguously a time separation and a space separation between x and x'? This question is equivalent to asking for the way in which reference frames inside a given field can be defined. A possible answer is provided by the definition of a reference frame given in Definition 8.

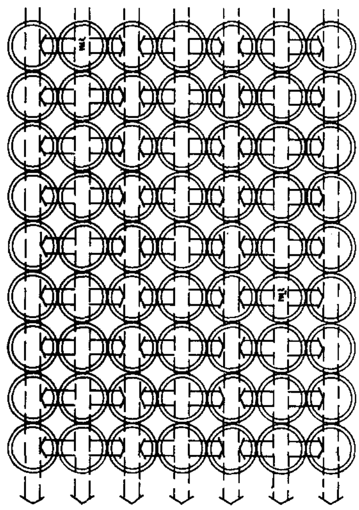

Fig. 6. We represent here a reference frame $\tilde{f}_g$ in a field f_g produced by a system S_g. Each double circle stands for a point (an equivalence class of partial complexes) of $\tilde{f}_g$. The vertical dotted double lines represent evolving channels. The horizontal dotted double lines stand for the immediate connections between evolving channels.

This definition is illustrated in Figure 6. Each double circle there stands for a point (an equivalence class of partial complexes) belonging to a partial field $\tilde{f}_g$ which is a reference frame in f_g. The vertical dotted double lines represent evolving channels. Each such evolving channel EC_i (i = 3–9) is made up of evolving particles entering in the partial complexes belonging to the points of $\tilde{f}_g$ which are in EC_i. The horizontal dotted double lines stand for the immediate connections between evolving channels.

We have already defined time and space separations between complexes belonging to the points of a given field. The extension of such definitions to the case of space and time separations between points of a partial field $\tilde{f}_g$ which is in turn a reference frame is straightforward. The time separation between two points $\tilde{x}$ and $\tilde{x}'$ of $\tilde{f}_g$ which are in the evolving channel EC_i is given by the number of points of $\tilde{f}_g$ found in EC_i between $\tilde{x}$ and $\tilde{x}'$. The space separation between $\tilde{y}, \tilde{y}' \in \tilde{f}_g$ is the number of points appearing on the same horizontal row between $\tilde{y}$ and $\tilde{y}'$. Notice that all of the consecutive points appearing on a horizontal row of Figure 6 are immediately connected, and that the sense of the direction of the connections alternate when we pass from one connection to the neighboring ones.

More generally, space and time separations can be unambiguously defined for any two points $\tilde{z}$ and $\tilde{z}'$ belonging to $\tilde{f}_g$. Suppose that $\tilde{z}$ and $\tilde{z}'$

appear both in different vertical and horizontal rows in Figure 6. Then, the space and time separations between $\tilde{z}$ and $\tilde{z}'$ is obtained by simply counting the corresponding intermediate vertical and horizontal rows. For instance, the space and time separations between $\tilde{z}$ and $\tilde{z}'$ in Figure 6 are 4 and 5.

In the particular case of $\tilde{f}_g$ (as we are considering in Figure 6) each evolving channel is immediately connected with either one or two different evolving channels. Therefore, the order of immediate connection of $\tilde{f}_g$ is 2. In addition, $\tilde{f}_g$ has a boundary of order 1. This corresponds to a reference frame with one bounded-space axis and one bounded-time axis. However, Definition 8 does not restrict the order of immediate connection to two, and therefore other examples corresponding to higher space-dimensionality can also be considered. For instance, a $\tilde{f}_s$ with an order of immediate connection equal to 6 and with no boundaries corresponds to a three-dimensional reference frame provided with a time axis.

All the α states of the particles entering into the partial complexes belonging to the points of $\tilde{f}_g$ are equivalent sets of preparticles [see point (ii) of Definition 8]. We then may ask for the origin of the nonsimilarity between partial complexes (see Definition 5) giving rise to different equivalence classes (points) belonging to $\tilde{f}_g$. This nonsimilarity arises as follows: (i) Partial complexes whose centers are the first α state of evolving particles immediately connecting points of $\tilde{f}_g$ are not similar to partial complexes whose centers are the last α state of such particles; (ii) for partial complexes in the same evolving channel EC_i we have that either the evolving particles belonging to EC_i are not similar, or there is a different number of evolving particles entering into each of these complexes; and (iii) for partial complexes in different evolving channels and to which the nonsimilarity mentioned in (i) does not apply, the nonsimilarity is of the same kind pointed out in (ii).

We have represented in Figure 6 only one reference frame, namely, the partial field $\tilde{f}_g$ of f_g. There may well exist other partial fields $\tilde{f}'_g, \tilde{f}''_g, \ldots,$ of f_g that are also reference frames in f_g. One of the conditions for this is that many evolving particles with varying number of α states and belonging to S_g connect the points of f_g.

Let us identify the points of $\tilde{f}_g$ by the centers of the partial complexes belonging to these points. These points are connected differently depending on which evolving particles we choose from the complexes $\sigma \in \sum \sum (S_g)$ to get the partial complexes. By varying this choice we find a redistribution of the double circles appearing on Figure 6 obtaining a new reference frame, which we call $\tilde{f}'_g$. This establishes a one-to-one relationship between the points of $\tilde{f}_g$ and the points of $\tilde{f}'_g$, although the space and time separations between the points of $\tilde{f}_g$ will differ from the separations between the points of $\tilde{f}'_g$. Both Galilean and Lorentzian transformations between two reference frames are particular cases of the above transformation between $\tilde{f}_g$ and $\tilde{f}'_g$. This is because both Galilean and Lorentzian transformations between two reference frames establish a one-to-one correspondence between the points of these reference frames (each point involving both the spatial and time coordinates). However, to reproduce such transformations in the present model would require a supplementary ad hoc restriction of the way in which, for instance, double circles of Figure 6 redistribute when we pass from $\tilde{f}_g$ to $\tilde{f}'_g$. Such a restriction will be studied in a forthcoming paper (García-Sucre, 1977).

How would a particle appear in a reference frame $\tilde{f}$? Suppose that the α state $s(x)$ of p has a nonempty intersection with the centers of some partial complexes belonging to the points $\tilde{x}, \tilde{x}', \ldots$ of a partial field $\tilde{f}$. Then we call *region of $\tilde{f}$ covered by $s(x)$* the subset $\tilde{f}_0$ of $\tilde{f}$ having as members just the points $\tilde{x}, \tilde{x}', \ldots$.

In the case of an evolving particle p, the regions $\tilde{f}_i, \tilde{f}_j, \ldots$ of a partial field $\tilde{f}$, respectively covered by the α states $s^i(x), s^j(y), \ldots$ of p can be ordered by the set $\sum(p)$. For instance, if $\sum(p) = (s^i(x), s^j(y), \ldots)$, then we say that the particle p first covers the region $\tilde{f}_i$ of $\tilde{f}$ and after the region $\tilde{f}_j$ of $\tilde{f}$, etc. The ordered set $(\tilde{f}_i, \tilde{f}_j, \ldots)$ of regions of $\tilde{f}$ is *the trajectory of p in $\tilde{f}$*. It may also occur that the intersection of some or all of the α states of p with every center of the partial complexes belonging to the points of $\tilde{f}$ is empty. In that case either some or all of the α states of p do not appear at any point of $\tilde{f}$.

A process can be viewed in the present model as follows: Think about an evolving particle p whose α states have all the same number of preparticles. Let the partial field $\tilde{f}$ be a reference frame defined in a field f. Consider, in addition, that the trajectory of p in $\tilde{f}$ is given by the ordered set $(\tilde{f}_1, \tilde{f}_2, \ldots)$. Then p first covers the region $\tilde{f}_1$ of $\tilde{f}$ and afterwards the region $\tilde{f}_2$ of $\tilde{f}$, and so on. Although the α states of p are equivalent sets of preparticles it may occur that the regions $\tilde{f}_1, \tilde{f}_2, \ldots$ of the reference frame $\tilde{f}$ present a very different aspect from each other. Some of them may be very localized in $\tilde{f}$—for instance, those α states whose regions in $\tilde{f}$ are sets of only one point of $\tilde{f}$, or a few neighboring points of $\tilde{f}$. Some other regions may be very delocalized in $\tilde{f}$, either by having many points of $\tilde{f}$ and/or by the distribution of its points in $\tilde{f}$ in a way as to have the appearance of an archipelago. An instance of process occurring to p in the above case would consist in the passage between two consecutive α states of p, say $s^i(x)$ and $s^j(y)$, such that $s^i(x)$ and $s^j(y)$ cover, respectively, a localized and a delocalized region in $\tilde{f}$.

The appearance of a trajectory of an evolving particle p' in a reference frame $\tilde{f}$ may be strongly discontinuous in the sense that successive points of the trajectory of p' may be greatly separated in $\tilde{f}$ both in space and time. However, for a system S having a sufficiently large number of certain types of evolving particles, it is always possible to select reference frames out of the field f produced by S so that the trajectory of p' looks continuous and simple.

In this connection (recalling Figure 6), when we pass from one reference frame to another the double circles in Figure 6 redistribute.

This feature of our model fits well with the fact that the conventions we currently use to define concrete reference frames are of such a nature that the trajectories of the particles observed look simple. A particular case of this is represented by the inertial reference frames in which the particles appear to be moving linearly and uniformly.

Notice that if S is a set of few particles then it is not possible to have regular extended entities as usual reference frames. In this case the reference frames may have only few points and therefore the motion of the particles may appear fragmentary.

Definition 9. Consider two physical systems S and S', and the fields $f = \sum\sum(S)/\backsim$, $f' = \sum\sum(S')/\backsim$ and $f'' = \sum\sum(SUS')/\backsim$. We say that S and S' interact, or are coupled, iff $f \cup f' \neq f''$. On the other hand, S and S' do not interact, or are uncoupled, iff $f \cup f = f''$.

By extension we say that two fields f and f' interact or are coupled when the two physical systems producing them interact. Similarly, for saying that f and f' do not interact, or are uncoupled, it is necessary that the two systems producing f and f' do not interact.

Let us now look at an example of the interaction between a system S of many particles and for instance a system $S_0 = \{p\}$ containing a unique particle p. Consider the field $f = \sum\sum(S)/\backsim$ and the regions of f covered by the α states of the particles belonging to S. Then, if we now consider the system $S' = S \cup \{p\}$, the simple addition of a particle p to S may produce a field $f' = \sum\sum(S')/\backsim$ different from f in the following three main respects.

(i) The number of points of f' may be different from those of f, since in the passage from f to f' either the similarity or dissimilarity between the complexes of f may be destroyed by adding a particle to some or all of the $\prod$ sets (see Definition 3) entering into the complexes belonging to the point of f.

(ii) Differences between the structure of the complexes belonging to the points of f' in relation to those belonging to the points of f.

(iii) Differences in the way the complexes of $\sum\sum(S')$ are connected in relation to the complexes of $\sum\sum(S)$.

The concept of interaction introduced in Definition 9 may be reconciled with the view of process exposed above by the following consideration: Suppose a field $f = \sum\sum(S)/\backsim$ and a subset $f_1 \subset f$. Then, it may happen that a particle of S, say p, does not appear in any complex belonging to the points of f_1. Accordingly, we could consider that f_1 is a field produced by a system $\tilde{S} \subset S$ such that $p \in S - \tilde{S}$. In this way, when an evolving particle p' passes throughout f_1 and penetrates in $f - f_1$, it could happen that some of the changes in the aspect presented by the regions covered by the α states of p'

may be attributed to the trait of $f - f_1$ arising by the presence of p in just this region of f.

From the points (i)–(iii) and the above consideration it may be understood how the interaction between systems of particle may change the geometry of the fields involved with or without change of topology.

3. SOME GENERAL PROPERTIES OF PARTICLES

In this section we start the study of some general properties of particles.

> *Theorem* 1. Given an evolving particle $p_m = \{a^m(x) \mid x \in X\}$, where $X \subseteq R$, with more than one α state, for $s^m(x_i), s^m(x_j) \in \sum(p_m)$ such that $s^m(x_i) \prec s^m(x_j)$, then $s^m(x_i) \cap s^m(x_j) = \varnothing$.

Proof. Taking into account equation (2.1), from $s^m(x_i), s^m(x_j) \in \sum(p_m)$ it follows that

$$s^m(x_i) = a^m(x_i) - \bigcup_{x_i' \in X(x_i)} a^m(x_i')$$

and

$$s^m(x_j) = a^m(x_j) - \bigcup_{x_j' \in X'(x_j)} a^m(x_j')$$

On the other hand, $s^m(x_i) \prec s^m(x_j)$ implies $a^m(x_i) \subset a^m(x_j)$. Thus

$$a^m(x_i) \subseteq \bigcup_{x_j' \in X'(x_j)} a^m(x_j')$$

and

$$[a^m(x_j) - \bigcup_{x_j' \in X'(x_j)} a^m(x_j')] \cap a^m(x_i) = \varnothing$$

i.e., $s^m(x_j) \cap a^m(x_i) = \varnothing$. Again, from equation (2.1) it follows that $s^m(x_i) \subseteq a^m(x_i)$, and therefore $s^m(x_j) \cap s^m(x_i) = \varnothing$.

As a consequence of the above property in the representations that we have given in Figures 1–6, there cannot be any evolving particles with any of their subsets being represented by closed curves. This implies that according to our definition of an evolving particle and α state of an evolving particle such particles cannot return to any of their α state. How can this fact be reconciled with the usual meaning of the state of a given particle according to which one can say, for instance, that a particle returns to its ground state? To answer this question, let us introduce a generalization of the concept of the state of a particle, which covers as particular cases the definition of α states given in equation (2.1), as well as the concept of state as is usually defined either in classical or quantum mechanics.

Definition 10. Let $\rho_a(p_i) \equiv \{\backsim_k \mid k \in K_i^a\}$, where $K_i^a \subset R$, be a set of equivalence relations well defined for all pair of α states of the particle p_i, and consider the quotient sets $\sum(p_i)/\backsim_k = \{\tilde{s}^i(\tilde{x}_k) \mid \tilde{x}_k \in \tilde{X}_k\}$, where $\tilde{X}_k \subset R$, for all $k \in K_i^a$. *We call a state of* p_i *with respect to* $\rho_a(p_i)$ any set $\tilde{s}^i(\tilde{x}) \equiv \bigcap_{k \in K_i^a} \tilde{s}^i(\tilde{x}_k)$, where a single element $\tilde{x}_k$ of $\tilde{X}_k$ occurs for each $k \in K_i^a$. For all the α states $s^i(x)$ of p_i such that $s^i(x) \in \tilde{s}^i(\tilde{x})$, we say that the particle p_i is in its $\tilde{s}^i(\tilde{x})$ state with respect to $\rho_a(p_i)$.

This definition reduces to the one given for α states in equation (2.1) in Section 2, in the case where $\rho_a(p_i)$ contains as unique element the following equivalence relation, which we denote by $\backsim_\alpha$: two states of p_i are equivalent if they contain the same preparticles. Definition 10 also covers the notion of a state of a particle in the current sense. In order to see this, let us assume that we have already defined what we understand to be the momentum of a particle in a given reference frame. Then let $\backsim_r$ and $\backsim_p$ be equivalence relations so that two states of the particle p_i are $\backsim_r$ and $\backsim_p$ equivalent if they correspond, respectively, to the same position and the same momentum of p_i in a given reference frame. In this case each set $\tilde{s}^i(\tilde{x}) = \bigcap_{k \in K_i^a} \tilde{s}^i(\tilde{x}_k) \neq \varnothing$, where $\rho_a(p_i) = \{\backsim_k \mid k \in K_i^a\}$ and $K_i^a = \{r, p\}$, corresponds to a state of a particle p_i in the Gibbs' phase space formalism.

Let us stress that, depending on the set $\rho_a(p_i)$, the physical meaning of a state of a particle may change drastically. An illustration of this fact is provided by the following example. Consider a particle p_i and the equivalence relations $\backsim_\alpha$ and $\backsim_r$, with the same meaning as above. To each equivalence class of the partition $\sum(p_i)/\backsim_\alpha$ belongs only one α state of p_i, and if p_i is an evolving particle, each state of p_i corresponds to a stage of its evolution to which the particle cannot return according to Theorem 1. On the other hand, each equivalence class of $\sum(p_i)/\backsim_r$ may contain more than one state, which according to Definition 10 corresponds to a particle p_i performing closed trajectories in a given reference frame.

Coming back again to the α states of a particle, let us say that even though an evolving particle cannot return to any of its α states (see Theorem 1 and Definition 10) a kind of return may occur between α states when more than one evolving particle is implicated. For instance, an evolving particle p_i can evolve in an opposite course with respect to another given evolving particle p_j in the sense that the members of the ordered sets $\sum(p_i)$ and $\sum(p_j)$, though identical, appear in an inverted order with respect to each other. Then, so to speak, two given evolving particles may be such that each of them penetrates in the past of the other. To give an example in which this occurs consider the two evolving particles $p_i = \{\{\alpha_1\}, \{\alpha_1, \alpha_2\}, \ldots, \{\alpha_1, \alpha_2, \ldots, \alpha_k\}, \{\alpha_1, \alpha_2, \ldots, \alpha_k, \alpha_l\}\}$ and $p_j = \{\{\alpha_l\}, \{\alpha_l, \alpha_k\}, \ldots, \{\alpha_l, \alpha_k, \ldots, \alpha_2\}, \{\alpha_l, \alpha_k, \ldots, \alpha_2, \alpha_1\}\}$, which are such that $\sum(p_i) = (\{\alpha_1\}, \{\alpha_2\}, \ldots, \{\alpha_k\}, \{\alpha_l\})$ and $\sum(p_j) = (\{\alpha_l\}, \{\alpha_k\}, \ldots, \{\alpha_2\}, \{\alpha_1\})$.

Theorem 2. Any state $s^i(x) \in \sum(p_i)$ of an evolving particle p_i is a nonempty set of preparticles.

Proof. Let $p_i = \{a^i(x) \mid x \in X\}$, where $X \subset R$, be an evolving particle. The elements $s^i(x)$ of $\sum(p_i)$ are given by equation (2.1) in Section 2:

$$s^i(x) = a^i(x) - \bigcup_{x' \in X'(x)} a^i(x') \tag{3.1}$$

where $a^i(x)$, $a^i(x') \in p_i$ and $a^i(x) \nsubseteq a^i(x')$. Since p_i is an evolving particle we have by definition that it must be a chain, i.e., the members of p_i can be placed in a chain of the type

$$\cdots a^i(x_r) \subset \cdots \subset a^i(x_s) \subset \cdots \subset a^i(x_t) \subset \cdots \tag{3.2}$$

where $\ldots, x_r, \ldots, x_s, \ldots, x_t, \ldots \in X$. In particular, the elements $a^i(x')$, where $x' \in X'(x)$, can be placed in a subchain of the above chain (3.2), namely,

$$\cdots \subset a^i(x_r') \subset \cdots \subset a^i(x_s') \subset \cdots \subset a^i(x_t') \subset \cdots \tag{3.3}$$

where $\cdots x_r', \ldots, x_s', \ldots, x_t', \ldots \in X'(x)$. Now, every member of p_i appearing either in chain (3.2) or chain (3.3) has the property of being identical to the union of itself with all the elements preceding it in the chain concerned. Then, in particular for the chain (3.3) we have

$$\bigcup_{x' \in \tilde{X}'(x)} a^i(\tilde{x}') = a^i(x') \tag{3.4}$$

for every $x' \in X'(x)$ and where $\tilde{X}(x')$ is such that $a^i(\tilde{x}')$ appears in the concerned chain and $a^i(\tilde{x}') \subseteq a^i(x')$.

Furthermore, in equation (3.1) we have $a^i(x) \nsubseteq a^i(x')$ for every $x' \in X'(x)$, which together with the chain (3.2) entails

$$a^i(x') \subset a^i(x) \tag{3.5}$$

for every $x' \in X'(x)$. Therefore, from equations (3.4) and (3.5) it follows that

$$\bigcup_{x' \in X'(x)} a^i(x') \subset a^i(x) \tag{3.6}$$

Inserting this result into equation (3.1) we finally obtain $s^i(x) \neq \varnothing$ for every $x \in X$.

> *Theorem* 3. An evolving particle is either a finite or denumerably infinite set.

Proof. By definition a particle is any subset of $P(B)$, where B is the set of all the preparticles. An evolving particle is a particular case of particle, and therefore any evolving particle p_i fulfills $p_i \subset P(B)$, where the proper inclusion stands, since $P(B)$ includes also nonevolving particles. In addition, the members of p_i are ordered by the proper inclusion relation $\subset$. Then, let p_i be given by the set $\{a^i(x) \mid x \in X\}$, where $X \subset R$. We shall prove that X must be equivalent to a subset of the set N of natural numbers.

To this end, let us consider the set $\sum(p_i)$ of α states of p_i. The members $s^i(x)$ of $\sum(p_i)$ are given by equation (2.1)

$$s^i(x) = a^i(x) - \bigcup_{x' \in X'(x)} a^i(x') \tag{3.7}$$

where $a^i(x)$, $a^i(x') \in p_i$ and $a^i(x) \nsubseteq a^i(x')$. It can be proved that $s^i(x) \neq s^i(y)$ if the associated members of p_i by equation (3.7) are different, namely, $a^i(x) \neq a^i(y)$, and conversely. Therefore, there exists a one-to-one mapping between $\sum(p_i)$ and p_i, i.e., $\sum(p_i) \leftrightarrow p_i$. To see this, let us recall that p_i is an evolving particle; and then, for any two different members $a^i(x)$ and $a^i(y)$ of p_i, one has from Theorem 1 that the corresponding α states $s^i(x)$ and $s^i(y)$ are disjoint sets, i.e., $s^i(x) \cap s^i(y) = \varnothing$. Accordingly, from $a^i(x) \neq a^i(y)$ it follows $s^i(x) \neq s^i(y)$.

Reciprocally, from $s^i(x) \neq s^i(y)$ and equation (3.7) it follows that

$$a^i(x) - \bigcup_{x' \in X'(x)} a^i(x') \neq a^i(y) - \bigcup_{y' \in X'(y)} a^i(y')$$

which together with either

$$\bigcup_{x' \in X'(x)} a^i(x') \subset a^i(x) \subseteq \bigcup_{y' \in X'(y)} a^i(y') \subset a^i(y)$$

or

$$\bigcup_{y' \in X'(y)} a^i(y') \subset a^i(y) \subseteq \bigcup_{x' \in X'(x)} a^i(x') \subset a^i(x)$$

yield $a^i(x) \neq a^i(y)$. In the above inclusion relations we have made use of equation (3.6) appearing in the proof of Theorem 2.

Accordingly, $s^i(x) \neq s^i(y)$ iff $a^i(x) \neq a^i(y)$ and therefore there exists a one-to-one mapping $\sum(p_i) \leftrightarrow p_i$.

Furthermore, since p_i is an evolving particle we have according to Theorem 2 concluded that none of the states $s^i(x) \in \sum(p_i)$ can be an empty set. Then we can introduce a mapping $\sum(p_i) \leftrightarrow \tilde{B}$ where $\tilde{B}$ is a subset of B containing only one preparticle of every $s^i(x) \in \sum(p_i)$, such that we make the unique preparticle belonging to $s^i(x) \bigcap \tilde{B}$ correspond to $s^i(x)$. The case $B = \tilde{B}$ occurs when p_i is an evolving particle such that the elements of $\sum(p_i)$ are sets of only one preparticle, and for every $\alpha_i \in B$ it follows that $\{\alpha_i\} \in \sum(p_i)$.

Now, all the elements of $\sum(p_i)$ are disjoint sets of preparticles (see Theorem 1), and accordingly the above mapping $\sum(p_i) \leftrightarrow \tilde{B}$ is such that different elements of $\sum(p_i)$ correspond to different elements of $\tilde{B}$.

Reciprocally, different members of $\tilde{B}$ correspond to different members of $\sum(p_i)$ since we have selected only one member of each $s^i(x) \in \sum(p_i)$ in order to put it in correspondence with $s^i(x)$ itself. We have thus the one-to-one mappings $p_i \leftrightarrow \sum(p_i)$ and $\sum(p_i) \leftrightarrow \tilde{B}$. We can write $p_i \leftrightarrow \tilde{B}$, which together with $\tilde{B} \subseteq B$, and the fact that B is a denumerably infinite set, entails that p_i is either finite or denumerably infinite.

> *Theorem 4.* A nonevolving particle is either a finite or an infinite set, and in the latter case can be either denumerably infinite or infinite with the cardinality of the continuum.

Proof. To see that a nonevolving particle can be a finite set it is sufficient to give an example as $\{\{\alpha_i\}, \{\alpha_j\}\}$, where $\alpha_i, \alpha_j \in B$, which without being a chain is, however, a nonempty subset of $P(B)$. In the same way, an instance of a nonevolving particle represented by a denumerably infinite set is provided by $\{\{\alpha_i\} \mid \alpha_i \in B\}$. Finally, let us show that there exist nonevolving particles such that the sets representing these particles have the cardinality N_1 of the continuum. To this end, let us consider the particle $p_i = P(B)$. From $p_i = P(B)$ it follows that there exist $a^i(x), a^i(y) \in p_i$ such that neither $a^i(x) \subset a^i(y)$ nor $a^i(x) \supset a^i(y)$. Accordingly, p_i, though a particle, is not an evolving particle. Also we have that, p_i is represented by a set whose cardinality is N_1, since $P(B)$ has the cardinality of the continuum provided that B is a denumerably infinite set. Any other particle $p_j \equiv P(B) - p_k$, where p_k is any evolving particle, equally provides an example of a nonevolving particle represented by a set whose cardinality is N_1, since any evolving particle is either a finite or a denumerably infinite set (see Theorem 3) and p_k does not remove from p_j elements such that neither $a^i(x) \subset a^i(y)$ nor $a^i(y) \subset a^i(x)$.

4. CONCLUDING REMARKS

Though the model of the physical world formulated above has a very general character (which is a typical deficiency of simple models) it presents the following positive features:

(i) The primitive concepts of the model are of a simple nature and intuitively clear, opening the possibility of full use of a fundamental mathematical formalism.

(ii) Particle, field, and interaction between systems of particles are obtained as derivative concepts.

(iii) An explicit relation is given between particles and fields (see Definitions 1–7).

(iv) The concept of field given here presents some general traits that are in agreement with recent ideas on the richness of the physics of the vacuum (Misner et al., 1970b). For instance, the points of a field and the way in which they are connected may have an intricate structure. Furthermore, the path of an evolving particle in a field may reveal fluctuations in the way in which the points of the field are connected.

(v) The model by its generality presents a flexible character sufficiently appropriate to make room for further developments in which descriptions of specific properties of physical systems could be made.

Finally, let us say that our model could be considered as an attempt to formulate a kind of pregeometry in the sense stated by Misner et al. (1970c), though the candidate of pregeometry that these authors tend to favor is the calculus of propositions.

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